PHYS 301
Electricity and Magnetism

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Today!

- Faraday's Law
- Maxwell's Law
- Maxwell's Equations

Faraday's Law

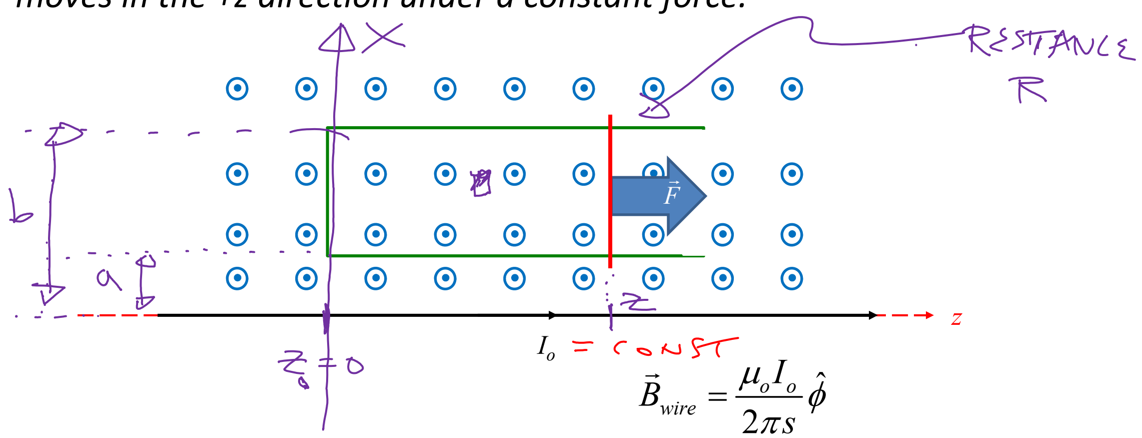
$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law

Current induced in loop near current carrying wire

- *Find induced current in green wire when red conducting "slider" moves in the +z direction under a constant force.*



$$\vec{B}_{wire} = \frac{\mu_o I_o}{2\pi s}\hat{\phi}$$

Faraday's Law

QUASISTATIC APPROXIMATION:

Changing $\vec{B}$'s produce $\vec{E}$'s, but often the formalism of magnetostatics is used to (approximately) describe the changing $\vec{B}$'s!

[~ okay, if $\vec{B}$ doesn't change "too rapidly!"]

e.g., fast enough to produce EM waves!

Faraday's Law

➢ **THINGS TO KEEP IN MIND …**

MOTIONAL EMFS - MAGNETIC FORCE SETS UP THE EMF, BUT DOES NOT TO WORK!

(it can't – why?!)

WHATEVER IS MOVING THE WIRES (LOOP — THIS FORCE IS TRANSMITTED TO CHARGES VIA STRUCTURE OF THE WIRE.

MAGNETIC INDUCTION — MAGNET MOVES (OR FIELD CHANGES) BUT LOOP IS STATIONARY; ∴ CHARGES ARE NOT MOVING SO NO MAGNETIC FORCE IS INVOLVED!

FORCE IS DUE TO AN ELECTRIC FIELD PRODUCED BY THE CHANGING MAGNETIC FIELD:

$$\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$$

❖ Fascinating that the two phenomena can be described in the same way.
As Einstein showed, it's the *relative motion* that is important.

Maxwell's Equations

So far, we have . . .

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_o \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \mu_o\vec{J}$$

★ Problem: $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V}) = 0 \quad \forall$ vectors $\vec{V}$

Consider Faraday's law . . .

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) \;=\; -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B}) \;=\; 0$$

Okay for both terms!

Maxwell's Equations

★ Problem: $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V}) = 0 \quad \forall$ vectors $\vec{V}$

Consider Ampere's law . . .

$$\underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})}_{=0} \;=\; \underbrace{\mu_o(\vec{\nabla}\cdot\vec{J})}_{\neq 0\text{, in general!}}$$

For steady currents,

$$\vec{\nabla}\cdot\vec{J} \;= 0 = -\frac{\partial\rho}{\partial t}$$

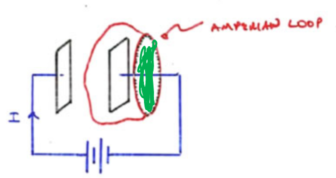
and it works . . .

But beyond this realm of magnetostatics it does not hold!

MAXWELL'S EQUATIONS

- **The standard example:**

Apply Ampere's law to the charging of a capacitor



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{ENCL} = \mu_o \int_{surf} \vec{J} \cdot d\vec{A}$$

★ Look at I_{ENCL} *through . . .*

- Surface in plane of LOOP:

$$I_{ENCL} = I$$

- Surface bounded by LOOP, but which passes between plates:

$$I_{ENCL} = 0$$

This was not a problem in magnetostatics because the currents were steady

Here, charge is piling up somewhere!!

(where?!)

MAXWELL'S EQUATIONS

★ **MAXWELL'S FIX:**

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0 = \mu_o (\vec{\nabla} \cdot \vec{J}) + \text{something!}$$

since $\quad \vec{\nabla} \cdot \vec{J} = -\dfrac{\partial \rho}{\partial t}$

and $\quad \rho = \varepsilon_o \vec{\nabla} \cdot \vec{E}$

then $\quad \vec{\nabla} \cdot \vec{J} = -\varepsilon_o \dfrac{\partial}{\partial t}\left(\vec{\nabla} \cdot \vec{E}\right) = -\vec{\nabla} \cdot \left[\varepsilon_o \dfrac{\partial \vec{E}}{\partial t}\right]$

So, let the **something** be $-\mu_o \vec{\nabla} \cdot \vec{J}$ written as $+\vec{\nabla} \cdot \left[\mu_o \varepsilon_o \dfrac{\partial \vec{E}}{\partial t}\right]$

Maxwell's Equations

THEN WE HAVE $$\vec{\nabla} \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

- Causes no trouble in magnetostatics, where $\vec{E} = \text{constant}$
- Implies that changing $\vec{E} \Rightarrow \vec{B}$
  Just like changing $\vec{B} \Rightarrow \vec{E}$ *(Faraday)*
- Crucial in propagation of waves
- Maxwell: $\vec{J}_d \equiv \varepsilon_o \frac{\partial \vec{E}}{\partial t} =$ DISPLACEMENT "CURRENT"

Maxwell's Equations in Free Space

$$\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_o \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Maxwell's Equations in Matter

$$\vec{\nabla}\cdot\vec{D} = \rho_f \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$$

where $\vec{D} = \varepsilon_o\vec{E} + \vec{P}$

$$\vec{H} = \frac{1}{\mu_o}\vec{B} - \vec{M}$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$